# UNIVERSITY OF DELHI
# DEPARTMENT OF MATHEMATICS
# B.A. (Programme)

**Learning Outcomes based Curriculum Framework (LOCF)**

**2019**

## Introduction

The modern citizen is routinely confronted by a maze of numbers and data of various forms in today's information-overload world. An increased knowledge of mathematics is essential to be able to make sense out of this. Mathematics is at the heart of many of today's advancements in economics, business, study of human behaviour, politics, science and technology. Studying mathematics along with social sciences can provide a firm foundation for further study in a variety of other disciplines. Students who have learned to logically question assertions, recognize patterns, and distinguish the essential and irrelevant aspects of problems can think deeply and precisely, nurture the products of their imagination to fruition, and share their ideas and insights

The design of the mathematical component in B.A. Programme seeks to balance a common intellectual foundation with opportunities to take advantage of the subject’s diverse applications and hence create the connections between mathematics and other humanistic disciplines.

## Learning outcomes of B.A. Programme:

A student opting for mathematics along with other humanity disciplines is able to:

- Solve problems using a broad range of significant mathematical techniques, including calculus, algebra, geometry, analysis, numerical methods, differential equations, probability and statistics along with hands-on-learning through CAS and LaTeX.
- Construct, modify and analyze mathematical models of systems encountered in disciplines such as economics, psychology, political sciences and sociology, assess the models' accuracy and usefulness, and draw contextual conclusions from them.
- Use mathematical, computational and statistical tools to detect patterns and model performance.
- Choose appropriate statistical methods and apply them in various data analysis problems.
- Use statistical software to perform data analysis.
- Have fundamental research design and mathematical/statistical skills needed to understand the acquired discipline specific knowledge.

## SEMESTER WISE PLACEMENT OF MATHS COURSES FOR B.A. (PROG.)

| **Sem-ester** | **Core Course (12)** | **Ability Enhancement Compulsory Course (AEC) (2)** | **Skill Enhancement Course (SEC) (4)** | **Discipline Specific Elective Course (DSE) (4)** |
|---|---|---|---|---|
| I | Calculus | | | |
| II | Algebra | | | |
| III | Analytic Geometry and Applied Algebra | | **SEC-1** Computer Algebra Systems | |
| IV | Analysis | | **SEC-2** Mathematical Typesetting System**:** LaTeX | |
| V | | | **SEC-3** Transportation and Network Flow Problems | **DSE-1** (i) Statistics OR (ii) Discrete Mathematics |
| VI | | | **SEC-4** Statistical Software**: R** | **DSE-2** (i) Numerical Methods OR (ii) Differential Equations |

# Semester-I

## Paper I: Calculus

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** Calculus is referred as 'Mathematics of change' and is concerned with describing the precise way in which changes in one variable relate to the changes in another. Through this course, students can understand the quantitative change in the behaviour of the variables and apply them on the problems related to the environment.

**Course Learning Outcomes:** The students who take this course will be able to:

i) Understand continuity and differentiability in terms of limits.
ii) Describe asymptotic behavior in terms of limits involving infinity.
iii) Use derivatives to explore the behavior of a given function, locating and classifying its extrema, and graphing the function.
iv) Understand the importance of mean value theorems.
v) Learn about Maclaurin's series expansion of elementary functions.

**Unit 1: Continuity and Differentiability of Functions**
Limits and Continuity, Types of discontinuities; Differentiability of functions, Successive differentiation, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

**Unit 2: Tracing of Curves**
Tangents and normals, Curvature, Singular points, Asymptotes, Tracing of curves.

**Unit 3: Mean Value Theorems and its Applications**
Rolle's theorem, Mean value theorems, Applications of mean value theorems to monotonic functions and inequalities; Taylor's theorem with Lagrange's and Cauchy's forms of remainder, Taylor's series, Maclaurin's series expansion of $e^x, \sin x, \cos x, \log(1+x)$ and $(1+x)^m$; Maxima and minima; Indeterminate forms.

**References:**
1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). Wiley India Pvt. Ltd. New Delhi. International Student Version. Indian Reprint 2016.
2. Prasad, Gorakh (2016). *Differential Calculus* (19th ed.). Pothishala Pvt. Ltd. Allahabad.

**Additional Reading:**
i. Thomas Jr., George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

**Teaching Plan (Paper-I: Calculus):**
**Weeks 1 and 2:** Limits and continuity, Types of discontinuities.
[1] Chapter 1 (Sections 1.1 to 1.6)
[2] Chapter 2 (Section 2.7).
**Week 3:** Differentiability of functions.
[1] Chapter 1 (Section 2.2).
**Week 4:** Successive differentiation, Leibnitz theorem. [2] Chapter 5.

**Week 5:** Partial differentiation, Euler's theorem on homogeneous functions.
[2] Sections 12.1 to 12.3.
**Week 6:** Tangents and normals.
[2] Chapter 8 (Sections 8.1 to 8.3).
**Week 7:** Curvature, Singular points.
[2] Chapter 10 (Sections 10.1 to 10.3, up to Page 224), and Chapter 11 (Sections 11.1 to 11.4).
**Weeks 8 and 9:** Asymptotes, Tracing of Curves.
[2] Chapter 9 (Sections 9.1 to 9.6), and Chapter 11 (Section 11.5).
**Weeks 10 and 11:** Rolle's theorem, Mean value theorems**:** Lagrange's mean value theorem, Cauchy's mean value theorem with geometrical interpretations, Applications of mean value theorems to monotonic functions and inequalities.
[2] Chapter 7 (Sections 7.4 to 7.6).
**Week 12:** Taylor's theorem with Lagrange's and Cauchy's forms of remainder, Taylor's series.
[2] Chapter 7 (Section 7.7).
**Week 13:** Maclaurin's series expansion of $e^x, \sin x, \cos x, \log(1+x)$, and $(1+x)^m$.
[2] Chapter 7 (Section 7.8).
**Week 14:** Maxima and minima; Indeterminate forms.
[2] Chapter 15 (Sections 15.1 to 15.3).
[1] Chapter 6 (Section 6.5).

### Facilitating the Achievement of Course Learning Outcomes

<table>
<tr><th>Unit No.</th><th>Course Learning Outcomes</th><th>Teaching and Learning Activity</th><th>Assessment Tasks</th></tr>
<tr><td>1.</td><td>Understand continuity and differentiability in terms of limits.</td><td rowspan="3">(i) Each topic to be explained with illustrations.<br>(ii) Students to be encouraged to discover the relevant concepts.<br>(iii) Students to be given homework/ assignments.<br>(iv) Discuss and solve the problems in the class.</td><td rowspan="3">• Presentations and class discussions.<br>• Assignments and class tests.<br>• Student presentations.<br>• Mid-term examinations.<br>• End-term examinations.</td></tr>
<tr><td>2.</td><td>Describe asymptotic behavior in terms of limits involving infinity.<br>Use derivatives to explore the behavior of a given function, locating and classifying its extrema, and graphing the function.</td></tr>
<tr><td>3.</td><td>Understand the importance of mean value theorems. Learn about Maclaurin's series expansion of elementary functions.</td></tr>
</table>

**Keywords:** Curvature, Euler's theorem on homogeneous functions, Leibnitz theorem, Maclaurin's theorem, Mean value theorems, Indeterminate forms Singular points and asymptotes, Tangents and normals, Taylor's series.

# Semester-II

## Paper II: Algebra

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** Students will get conceptual understanding and the applicability of the subject matter. helps students to see how linear algebra can be applied to real-life situations. Modern concepts and notation are used to introduce the various aspects of linear equations, leading readers easily to numerical computations and applications.

**Course Learning Outcomes:** The course will enable the students to understand:

i) Solving higher order algebraic equations.
ii) Become aware of De Moivre's theorem and its applications.
iii) Solving simultaneous linear equations with at most four unknowns.
iv) Get an overview of abstract algebra by learning about algebraic structures namely, groups, rings and vector spaces.

**Unit 1: Theory of Equations and Expansions of Trigonometric Functions**
Fundamental Theorem of Algebra, Relation between roots and coefficients of $n$th degree equation, Remainder and factor theorem, Solutions of cubic and biquadratic equations, when some conditions on roots of the equation are given, Symmetric functions of the roots for cubic and biquadratic; De Moivre's theorem (both integral and rational index), Solutions of equations using trigonometry and De Moivre's theorem, Expansion for $\cos nx, \sin nx$ in terms of powers of $\cos x$ , $\sin x$, and $\cos^n x$ , $\sin^n x$, in terms of cosine and sine of multiples of $x$.

**Unit 2: Matrices**
Matrices, Types of matrices, Rank of a matrix, Invariance of rank under elementary transformations, Reduction to normal form, Solutions of linear homogeneous and non-homogeneous equations with number of equations and unknowns up to four; Cayley–Hamilton theorem, Characteristic roots and vectors.

**Unit 3: Groups, Rings and Vector Spaces**
Integers modulo $n$, Permutations, Groups, Subgroups, Lagrange's theorem, Euler's theorem, Symmetry Groups of a segment of a line, and regular $n$-gons for $n$ = 3, 4, 5, and 6; Rings and subrings in the context of C[0,1] and $\mathbb{Z}_n$; Definition and examples of a vector space, Subspace and its properties, Linear independence, Basis and dimension of a vector space.

**References:**

1. Beachy, J. A., & Blair, W. D. (2006). *Abstract Algebra* (3rd ed.). Waveland Press, Inc.
2. Burnside, William Snow (1979). *The Theory of Equations*, Vol. 1 (11th ed.) S. Chand & Co. Delhi. Fourth Indian Reprint.
3. Gilbert, William J., & Vanstone, Scott A. (1993). *Classical Algebra* (3rd ed.). Waterloo Mathematics Foundation, Canada.
4. Meyer, Carl D. (2000). *Matrix Analysis and Applied Linear Algebra*. Society for Industrial and Applied Mathematics (Siam).

**Additional Readings:**

i. Dickson, Leonard Eugene (2009). *First Course in The Theory of Equations*. The Project

Gutenberg EBook (http://www.gutenberg.org/ebooks/29785).
ii. Gilbert, William J. (2004). *Modern Algebra with Applications* (2nd ed.). Wiley-Interscience, John Wiley & Sons.

**Teaching Plan (Paper-II: Algebra):**

**Weeks 1 and 2:** Fundamental Theorem of Algebra (statement only), Relation between roots and coefficients of $n$th degree equation, Remainder and Factor Theorem, Solutions of cubic and biquadratic equations, when some conditions on roots of the equation are given.
[2] Chapter 3.

**Week 3:** Symmetric functions of the roots for cubic and biquadratic equations.
[2] Chapter 4.

**Weeks 4 and 5:** De Moivre's theorem (both integral and rational index), Solutions of equations using trigonometry and De Moivre's theorem, Expansion for $\cos nx, \sin nx$ in terms of powers of $\cos x, \sin x$, and $\cos^n x, \sin^n x$, in terms of cosine and sine of multiples of $x$.
[3] Sections 7.6, and 7.7.

**Week 6:** Matrices, Types of matrices, Introduction elementary transformations.
[4] Chapter 3 (Sections 3.2, 3.5, and 3.7)

**Week 7:** Rank of a matrix. Invariance of rank under elementary transformations.
[4] Section 3.9.

**Week 8:** Reduction to normal (Echelon) form, Solutions of linear homogeneous and non-homogeneous equations with number of equations and unknowns up to four.
[4] Chapter 2 (Sections 2.1 to 2.5).

**Week 9:** Cayley–Hamilton theorem, Characteristic roots and vectors.
[4] Chapter 7 (Section 7.1, and Example 7.2.2)

**Week 10:** Integers modulo $n$, Permutations.
[1] Chapter 1 (Section 1.4), and Chapter 2 (Section 2.3).

**Week 11:** Groups, subgroups, Examples of groups, subgroups and simple theorems.
[1] Chapter 3 (Sections 3.1, and 3.2)

**Week 12:** Lagrange's theorem, Euler's theorem, Symmetry groups of a segment of a line, and regular $n$-gons for $n$ = 3, 4, 5 and 6; Rings and subrings in the context of C[0,1] and $\mathbb{Z}_n$.
[1] Chapter 3 (Sections 3.2, 3.3, and 3.6), and Chapter 5 (Section 5.1)

**Weeks 13 and 14:** Definition and examples of vector space, Subspace and its properties, Linear independence, Basis and dimension of a vector pace.
[4] Chapter 4 (Sections 4.1, 4.3, and 4.4).

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Solving higher order algebraic equations.<br>Become aware of De Moivre's theorem and its applications. | (i) Each topic to be explained with examples.<br>(ii) Students to be involved in discussions and encouraged to ask questions.<br>(iii) Students to be given homework/assignments.<br>(iv) Students to be encouraged to give short presentations. | • Student presentations.<br>• Participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• End-term examinations. |
| 2. | Solving simultaneous linear equations with at most four unknowns. | | |
| 3. | Get an overview of abstract algebra by learning about algebraic structures namely, groups, rings and vector spaces. | | |

**Keywords:** Basis and dimension of vector space, Cayley–Hamilton theorem, Characteristic roots and vectors, Fundamental theorem of algebra, Linear dependence and independence, Lagrange's theorem, Permutations, Rank of a matrix.

# Semester-III

## Paper III: Analytic Geometry and Applied Algebra

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** The course aims at identifying curves and applying mathematical models in daily life problems, studying geometric properties of various conic sections. The purpose of this course is to strengthen the mathematical skills along with the algebraic skills and concepts to assure success in the algebra.

**Course Learning Outcomes:** The course will enable the students to:

i) Learn concepts in two-dimensional geometry.
ii) Identify and sketch conics namely, ellipse, parabola and hyperbola.
iii) Learn about three-dimensional objects such as spheres, conicoids, straight lines and planes using vectors.
iv) Understand various applications of algebra in design of experiments, modelling of matching jobs, checking spellings, network reliability and scheduling of meetings.

**Unit 1: Geometry**
Techniques for sketching parabola, ellipse and hyperbola, Reflection properties of parabola, ellipse, hyperbola and their applications to signals, Classification of quadratic equations representing lines, parabola, ellipse and hyperbola.

**Unit 2: 3-Dimensional Geometry and Vectors**
Rectangular coordinates in 3-dimensional space, Spheres, Cylindrical surfaces, Cones, Vectors viewed geometrically, Vectors in coordinate systems, Vectors determined by length and angle, Dot product, Cross product and their geometrical properties, Parametric equations of lines in plane, Planes in 3-dimensional space.

**Unit 3: Applied Algebra**
Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments, Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings, Interval graph modelling and influence model, Pitcher pouring puzzle.

**References:**

1. Anton, Howard; Bivens, Irl & Davis, Stephen (2013). *Calculus* (10th ed.). Wiley India Pvt. Ltd. New Delhi. International Student Version. India. Reprint 2016.
2. Gulberg, Jan. (1997). *Mathematics from the Birth of Numbers*. W.W. Norton & Co.
3. Tucker, Alan (2012). *Applied Combinatorics* (6th ed.). John Wiley & Sons, Inc.

**Additional Reading:**

i. Lidl, Rudolf & Pilz, Günter (1998). *Applied Abstract Algebra* (2nd ed.). Springer. Indian Reprint 2014.

**Teaching Plan (Paper III: Analytic Geometry and Applied Algebra):**
**Weeks 1 to 3:** Techniques for sketching parabola, ellipse and hyperbola with problem solving.
[1] Chapter 11 (Section 11.4).

**Weeks 4 and 5:** Reflection properties of parabola, ellipse and hyperbola, Classification of quadratic equation representing lines, parabola, ellipse and hyperbola, Rotation of axis second degree equations

[1] Chapter 11 (Sections 11.4, and 11.5).

**Weeks 6 and 7:** Rectangular coordinates in 3-dimensional space with problems, Spheres, Cylindrical surfaces, Cones.

[1] Chapter 12 (Section 12.1).

**Weeks 8 and 9:** Vectors in coordinate systems, Vectors viewed geometrically, Vectors determined by length and angle, Dot product, Cross product and their geometrical properties.

[1] Chapter 12 (Sections 12.3, and 12.4).

**Weeks 10 and 11:** Parametric equations of lines in plane, Planes in 3-dimensional space.

[1] Chapter 12 (Sections 12.4, 12.5).

**Weeks 12 to 14:** Latin squares, Table for a finite group as a Latin square, Latin squares as in design of experiments, Mathematical models for matching jobs, Spelling checker, Network reliability, Street surveillance, Scheduling meetings. Interval graph modelling and Influence model, Pitcher pouring puzzle.

[2] Chapter 5 (Page 195).

[3] Chapter 1 (Section 1.1, Examples 1 to 6), and Chapter 3 (Section 3.2, Example 3, Page 106).

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Learn concepts in two-dimensional geometry. Identify and sketch conics namely, ellipse, parabola and hyperbola. | (i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations. | • Student presentations. • Participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations. |
| 2. | Learn about three-dimensional objects such as spheres, conicoids, straight lines and planes using vectors. | | |
| 3. | Understand various applications of algebra in design of experiments, modelling of matching jobs, checking spellings, network reliability and scheduling of meetings. | | |

**Keywords:** Latin squares, Parabola, Ellipse, Hyperbola, Pitcher pouring puzzle, Spelling checker.

# Skill Enhancement Paper

## SEC-1: Computer Algebra Systems

**Total Marks: 100** (Theory: 38, Internal Assessment: 12, and Practical: 50)
**Workload:** 2 Lectures, 4 Practicals (per week) **Credits:** 4 (2+2)
**Duration:** 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

**Course Objectives:** This course aims at providing basic knowledge to Computer Algebra Systems (CAS) and their programming language in order to apply them for plotting functions, finding roots to polynomials, computing limits and other mathematical tools.

**Course Learning Outcomes:** This course will enable the students to:

i) Use CAS as a calculator and for plotting functions.
ii) Understand the role of CAS finding roots of polynomials and solving general equations.
iii) Employ CAS for computing limits, derivatives, and computing definite and indefinite integrals.
iv) Use CAS to understand matrix operations and to find eigenvalues of matrices.

**Unit 1: Introduction to CAS and Graphics**
Computer Algebra Systems (CAS), Use of a CAS as a calculator, Simple programming in a CAS; Computing and plotting functions in 2D, Customizing plots, Animating plots; Producing table of values, Working with piecewise defined functions, Combining graphics.

**Unit 2: Applications in Algebra**
Factoring, Expanding and finding roots of polynomials, Working with rational and trigonometric functions, Solving general equations.

**Unit 3: Applications of Calculus**
Computing limits, First and higher order derivatives, Maxima and minima, Integration, Computing definite and indefinite integrals.

**Unit 4: Working with Matrices**
Performing Gaussian elimination, Operations (transpose, determinant, and inverse), Minors and cofactors, Solving systems of linear equations, Rank and nullity of a matrix, Eigenvalue, eigenvector and diagonalization.

**References:**

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Torrence, Bruce F., & Torrence, Eve A. (2009). *The Student's Introduction to Mathematica®: A Handbook for Precalculus, Calculus, and Linear Algebra* (2nd ed.). Cambridge University Press.

**Note**: Theoretical and Practical demonstration should be carried out ***only in one*** of the CAS**:** Mathematica/MATLAB/Maple/Maxima/Scilab or any other.

## Practicals to be done in the Computer Lab using CAS Software:

[1] Chapter 12 (Exercises 1 to 4 and 8 to 12).
[2] Chapter 3 [Exercises 3.2 (1), 3.3 (1, 2 and 4), 3.4 (1 and 2), 3.5 (1 to 4), 3.6 (2 and 3)].
[2] Chapter 4 (Exercises 4.1, 4.2, 4.5, 4.7 and 4.9).

[2] Chapter 5 [Exercises 5.1 (1), 5.3, 5.5, 5.6 (1, 2 and 4), 5.10 (1 and 3), 5.11 (1 and 2)].
[2] Chapter 7 [Exercises 7.1 (1), 7.2, 7.3 (2), 7.4 (1) and 7.6].

### Teaching Plan (Theory of SEC-1: Computer Algebra Systems):

**Weeks 1 and 2:** Computer Algebra Systems (CAS), Use of a CAS as a calculator, Simple programming in a CAS.

[1] Chapter 12 (Sections 12.1 to 12.5).

**Weeks 3 to 5:** Computing and plotting functions in 2D, Customizing plots, Animating plots, Producing table of values, Working with piecewise defined functions, Combining graphics.

[2] Chapter 1, Chapter 3 (Sections 3.1 to 3.6, and 3.8)

**Weeks 6 to 8:** Factoring, Expanding and finding roots of polynomials, Working with rational and trigonometric functions, Solving general equations.

[2] Sections 4.1 to 4.3, 4.5 to 4.7, and 4.9.

**Weeks 9 to 11:** Computing limits, First and higher order derivatives, Maxima and minima, Integration, computing definite and indefinite integrals.

[2] Chapter 5 (Sections 5.1, 5.3, 5.5, 5.6, 5.10, and 5.11).

**Weeks 12 to 14:** Performing Gaussian elimination, Operations (transpose, determinant, and inverse), Minors and cofactors, Solving systems of linear equations, Rank and nullity of a matrix, Eigenvalue, eigenvector and diagonalization.

[2] Chapter 7 (Sections 7.1 to 7.4, and 7.6 to 7.8).

### Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Use CAS as a calculator and for plotting functions. | (i) Each topic to be explained with illustrations and using CAS.<br>(ii) Students to be given homework/assignments.<br>(iii) Students to be encouraged to look for new applications. | • Presentations and class discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• Practical examinations.<br>• End-term examinations. |
| 2. | Understand the role of CAS finding roots of polynomials and solving general equations. | | |
| 3. | Employ CAS for computing limits, derivatives, and computing definite and indefinite integrals. | | |
| 4. | Use CAS to understand matrix operations and to find eigenvalues of matrices. | | |

**Keywords:** Computer Algebra Systems (CAS), CAS in graphics, CAS in algebra, CAS in calculus.

# Semester-IV

## Paper IV: Analysis

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** The course aims at building an understanding of convergence of sequence and series of real numbers and various methods/tools to test their convergence. The course also aims at building understanding of the theory of Riemann integration.

**Course Learning Outcomes:** The course will enable the students to:
i) Understand basic properties of the field of real numbers.
ii) Examine continuity and uniform continuity of functions using sequential criterion.
iii) Test convergence of sequence and series of real numbers.
iv) Distinguish between the notion of integral as anti-derivative and Riemann integral.

**Unit 1: Real numbers and Real Valued Functions**
Algebraic and order properties of $\mathbb{R}$, Absolute value and the real line, Suprema and infima, The completeness and Archimedean property of $\mathbb{R}$; Limit of functions, Sequential criterion for limits, Algebra of limits, Continuous functions, Sequential criterion for continuity and discontinuity, Properties of continuous functions, Uniform continuity.

**Unit 2: Sequence and Series**
Sequences and their limits, Convergent sequences, Limit theorems, Monotone sequences and their convergence, Subsequences, Cauchy sequence and convergence criterion; Infinite series and their convergence, Cauchy criterion for series, Positive term series, Comparison tests, Absolute and conditional convergence, Cauchy's $n$th root test, D'Alembert's ratio test, Raabe's test, Alternating series, Leibnitz test.

**Unit 3: Riemann Integral**
Riemann integral, Integrability of continuous and monotonic functions.

**References:**

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition.
2. Ross, Kenneth A. (2013). *Elementary Analysis*: *The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.


**Additional Readings:**

i. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
ii. Denlinger, Charles G. (2011). *Elements of Real Analysis*. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.


**Teaching Plan (Paper IV: Analysis):**
**Week 1:** Algebraic and order properties of $\mathbb{R}$, Absolute value and the real line.
[1] Chapter 2 (Sections 2.1 and 2.2)
**Weeks 2 and 3:** Suprema and infima**,** The completeness properties of $\mathbb{R}$, Archimedean property of $\mathbb{R}$.
[1] Chapter 2 (Sections 2.3 and 2.4).

**Weeks 4 and 5:** Sequences and their limits, Convergent sequences, Limit theorems.
[1] Chapter 3 (Sections 3.1 and 3.2).
**Week 6:** Monotone sequences and monotone convergence theorem.
[1] Chapter 3 (Section 3.3).
**Week 7:** Subsequences, Cauchy sequence and Cauchy convergence criterion.
[1] Chapter 3 (Sections 3.4 [3.4.1, 3.4.2, 3.4.3, 3.4.5, 3.4.6{(a), (b)}, 3.4.8 (Statement only) and 3.5 [up to 3.5.6]).
**Weeks 8 and 9:** Infinite series, Convergence of a series, *n*th term test, Cauchy's criterion for series, The *p*-series, Positive term series, Comparison tests, Absolute and conditional convergence.
[1] Chapter 3 (Section 3.7), Chapter 9 [Section 9.1 (9.1.1 and 9.1.2)].
**Week 10:** Cauchy's *n*th root test, D'Alembert's ratio test, Raabe's test, Alternating series, Leibnitz test.
[1] Chapter 9 [Sections 9.2 (Statements of tests only) and 9.3 (9.3.1 and 9.3.2)].
**Week 11:** Limit of functions, Sequential criterion for limits, Algebra of limits.
[1] Chapter 4 (Sections 4.1 and 4.2).
**Week 12:** Continuous functions, Sequential criterion for continuity and discontinuity, Boundedness theorem, Intermediate value theorem, Uniform continuity.
[1] Chapter 5 (Sections 5.1, 5.3, and 5.4 excluding continuous extension and approximation)
**Week 13:** Riemann integral**:** Upper and lower integrals, Riemann integrable functions.
[2] Chapter 6 (Section 32, only statement of the results up to Page 274, with Examples 1, and 2)
**Week 14:** Riemann integrability of continuous and monotone functions.
[2] Chapter 6 [Section 33 (33.1 and 33.2)].

**Facilitating the Achievement of Course Learning Outcomes**

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Understand basic properties of the field of real numbers. Examine continuity and uniform continuity of functions using sequential criterion. | (i) Each topic to be explained with examples.<br>(ii) Students to be involved in discussions and encouraged to ask questions.<br>(iii) Students to be given homework/assignments.<br>(iv) Students to be encouraged to give short presentations. | • Student presentations.<br>• Participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• End-term examinations. |
| 2. | Test convergence of sequence and series of real numbers. | | |
| 3. | Distinguish between the notion of integral as anti-derivative and Riemann integral. | | |

**Keywords:** Continuity, Cauchy convergence criterion, Convergence, Cauchy's *n*th root test, D'Alembert's ratio test, Intermediate value theorem, Riemann integral, Supremum, Uniform continuity.

# Skill Enhancement Paper

## SEC-2: Mathematical Typesetting System: LaTeX

**Total Marks: 100** (Theory: 38, Internal Assessment: 12, and Practical: 50)
**Workload:** 2 Lectures, 4 Practicals (per week) **Credits:** 4 (2+2)
**Duration:** 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

**Course Objectives:** The purpose of this course is to help you begin using LaTeX, a mathematical typesetting system designed for the creation of beautiful books – and especially for books that contain a lot of mathematics, complicated symbols and formatting.

**Course Learning Outcomes:** This course will enable the students to:

i) Create and typeset a LaTeX document.
ii) Typeset a mathematical document using LaTex.
iii) Learn about pictures and graphics in LaTex.
iv) Create beamer presentations.

**Unit 1: Getting Started with LaTeX**
Introduction to TeX and LaTeX, Creating and typesetting a simple LaTeX document, Adding basic information to documents, Environments, Footnotes, Sectioning, Displayed material.

**Unit 2: Mathematical Typesetting**
Accents and symbols; Mathematical typesetting (elementary and advanced): Subscript/ Superscript, Fractions, Roots, Ellipsis, Mathematical symbols, Arrays, Delimiters, Multiline formulas, Putting one thing above another, Spacing and changing style in math mode.

**Unit 3: Graphics and PSTricks**
Pictures and graphics in LaTeX, Simple pictures using PSTricks, Plotting of functions.

**Unit 4: Getting Started with Beamer**
Beamer, Frames, Setting up beamer document, Enhancing beamer presentation.

**References:**

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Lamport, Leslie (1994). *LaTeX: A Document Preparation System*, User's Guide and Reference Manual (2nd ed.). Pearson Education. Indian Reprint.

**Additional Reading:**

i. Dongen, M. R. C. van (2012). *LaTeX and Friends*. Springer-Verlag.

## Practicals to be done in the Computer Lab using a suitable LaTeX Editor:

[1] Chapter 9 (Exercises 4 to 10), Chapter 10 (Exercises 1, 3, 4, and 6 to 9), and Chapter 11 Exercises 1, 3, 4, 5).

**Teaching Plan (Theory of SEC-2: Mathematical Typesetting System: LaTeX):**
**Weeks 1 to 3:** Introduction to TeX and LaTeX, Creating and typesetting a simple LaTeX document, adding basic information to documents, Environments, Footnotes, Sectioning, Displayed material.
[1] Chapter 9 (Sections 9.1 to 9.5).
[2] Chapter 2 (Sections 2.1 to 2.5).

**Weeks 4 to 7:** Accents and symbols; Mathematical typesetting (elementary and advanced)**:** Subscript/Superscript, Fractions, Roots, Ellipsis, Mathematical symbols, Arrays, Delimiters, Multiline formulas, Putting one thing above another, Spacing and changing style in math mode.

[1] Chapter 9 (Sections 9.6, and 9.7).

[2] Chapter 3 (Sections 3.1 to 3.3).

**Weeks 8 to 11:** Pictures and graphics in LaTeX, Simple pictures using PS Tricks, Plotting of functions.

[1] Chapter 9 (Section 9.8), and Chapter 10 (Sections 10.1 to 10.3)

[2] Chapter 7 (Sections 7.1, and 7.2)

**Weeks 12 to 14:** Beamer, Frames, Setting up beamer document, Enhancing beamer presentation.

[1] Chapter 11 (Sections 11.1 to 11.4)

## Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Create and typeset a LaTeX document. | (i) Topics to be explained using LaTex editor.<br>(ii) Students to be given homework/assignments.<br>(iii) Students to be encouraged to look for new applications. | • Presentations and class discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• Practical examinations.<br>• End-term examinations. |
| 2. | Typeset a mathematical document using LaTex. | | |
| 3. | Learn about pictures and graphics in LaTex. | | |
| 4. | Create beamer presentations. | | |

**Keywords:** LaTex, Mathematical typesetting, PSTricks, Beamer.

# Semester-V

## Skill Enhancement Paper

### SEC-3: Transportation and Network Flow Problems

**Total Marks: 100** (Theory: 55, Internal Assessment: 20, and Practical: 25)
**Workload:** 3 Lectures, 2 Practicals (per week) **Credits:** 4 (3+1)
**Duration:** 14 Weeks (42 Hrs. Theory + 28 Hrs. Practical) **Examination: 3** Hrs.

**Course Objectives:** This course aims at providing applications of linear programming to solve real-life problems such as transportation problem, assignment problem, shortest-path problem, minimum spanning tree problem, maximum flow problem and minimum cost flow problem.

**Course Learning Outcomes:** This course will enable the students to:

i) Formulate and solve transportation problems.
ii) Learn to solve assignment problems using Hungarian method.
iii) Solve travelling salesman problem.
iv) Learn about network models and various network flow problems.
v) Learn about project planning techniques namely, CPM and PERT.

**Unit 1: Transportation Problems**
Transportation problem and its mathematical formulation, North West corner method, Least cost method and Vogel's approximation method for determination of starting basic feasible solution, Algorithm for solving transportation problem.

**Unit 2: Assignment and Traveling Salesperson Problems**
Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem, Traveling salesperson problem.

**Unit 3: Network Models**
Network models, Minimum spanning tree algorithm, Shortest-route problem, Maximum flow model.

**Unit 4: Project Management with CPM/PERT**
Project network representation, CPM and PERT.

**References:**

1. Hillier, Frederick S., & Lieberman, Gerald J. (2017). *Introduction to Operations Research* (10th ed.). McGraw Hill Education (India) Pvt. Ltd. New Delhi.
2. Taha, Hamdy A. (2007). *Operations Research*: *An Introduction* (8th ed.). Pearson Education India. New Delhi.

**Additional Reading:**

i. Bazaraa, Mokhtar S., Jarvis, John J., & Sherali, Hanif D. (2010). *Linear Programming and Network Flows* (4th ed.). John Wiley & Sons.

## Practicals to be done in the Computer Lab using a suitable Software:

Use TORA/Excel spreadsheet to solve transportation problem, Assignment problem, Traveling salesperson problem, Shortest-route problem, Minimum spanning tree algorithm, Maximum flow model, CPM and PERT calculations of exercises from the Chapters 5 and 6 of [2].

[1] Case 9.1: Shipping Wood to Market, and Case 9.3: Project Pickings.

**Teaching Plan (Theory of SEC-3: Transportation and Network Flow Problems):**

**Weeks 1 to 4:** Transportation problem and its mathematical formulation, North West corner method, least cost method and Vogel's approximation method for determination of starting basic feasible solution. Algorithm for solving transportation problem.

[2] Chapter 5 (Sections 5.1, and 5.3).

**Weeks 5 to 7:** Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem, traveling salesperson problem.

[2] Sections 5.4, and 9.3.

**Weeks 8 to 11:** Network models, minimum spanning tree algorithm, shortest-route problem, maximum flow model.

[2] Chapter 6 (Sections 6.1 to 6.4).

**Weeks 12 to 14:** Project network, CPM and PERT.

[2] Chapter 6 (Section 6.5).

**Facilitating the Achievement of Course Learning Outcomes**

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Formulate and solve transportation problems. | (i) Topics to be explained with illustrations using TORA/Excel.<br>(ii) Students to be given homework/assignments.<br>(iii) Students to be encouraged to look for new applications. | • Presentations and class discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• Practical examinations.<br>• End-term examinations. |
| 2. | Learn to solve assignment problems using Hungarian method.<br>Solve travelling salesman problem. | | |
| 3. | Learn about network models and various network flow problems. | | |
| 4. | Learn about project planning techniques namely, CPM and PERT. | | |

**Keywords:** Transportation problem, Assignment problem, Traveling salesperson problem, Network flows, CPM, PERT.

# Mathematics: Discipline Specific Elective (DSE) Course -1

**Any *one* of the following:**
**DSE-1 (i): Statistics**
**DSE-1 (ii): Discrete Mathematics**

## DSE-1 (i): Statistics

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** The course aims at building a strong foundation of theory of statistical distributions as well as understanding some of the most commonly used distributions. The course also aims to equip the students to analyze, interpret and draw conclusions from the given data.

**Course Learning Outcomes:** The course will enable the students to:

i) Determine moments and distribution function using moment generating functions.
ii) Learn about various discrete and continuous probability distributions.
iii) Know about correlation and regression for two variables, weak law of large numbers and central limit theorem.
iv) Test validity of hypothesis, using Chi-square, F and t-tests, respectively in sampling distributions.

**Unit 1: Probability, Random Variables and Distribution Functions**
Sample space, Events, Probability Classical, Relative frequency and axiomatic approaches to probability, Theorems of total and compound probability; Conditional probability, Independent events, Baye's Theorem; Random variables (discrete and continuous), Probability distribution, Expectation of a random variable, Moments, Moment generating functions.

**Unit 2: Discrete and Continuous Probability Distributions**
Discrete and continuous distribution, Binomial, Poisson, Geometric, Normal and exponential distributions, Bivariate distribution, Conditional distribution and marginal distribution, Covariance, Correlation and regression for two variables, Weak law of large numbers and central limit theorem for independent and identically distributed random variables.

**Unit 3: Sampling Distributions**
Statistical inference**:** Definitions of random sample, Parameter and statistic, Sampling distribution of mean, Standard error of sample mean; Mean, variance of random sample from a normal population; Mean, variance of random sample from a finite population; Chi-square distribution, F distribution and t distribution, Test of hypotheses based on a single sample.

**References:**

1. Devore, Jay L., & Berk, Kenneth N. (2007). *Modern Mathematical Statistics with Applications*. Thomson Brooks/Cole.
2. Miller, Irvin & Miller, Marylees (2006). John E. Freund's: *Mathematical Statistics with Applications* (7th ed.). Pearson Education, Asia.

**Additional Readings:**


i. Hayter, Anthony (2012). *Probability and Statistics for the Engineers and Scientists* (4th ed.). Brooks/Cole, Cengage Learning.
ii. Mood, Alexander M., Graybill, Franklin A., & Boes, Duane C. (1974). *Introduction to the Theory of Statistics* (3rd ed.). McGraw-Hill Inc. Indian Reprint 2017.
iii. Rohtagi, Vijay K., & Saleh, A. K. Md. E. (2001). *An Introduction to Probability and Statistics* (2nd ed.). John Wiley & Sons, Inc. Wiley India Edition 2009.


**Teaching Plan (DSE-1 (i): Statistics):**

**Week 1:** Sample space, Events, Probability Classical, Relative frequency and axiomatic approaches to probability, Theorems of total and compound probability.
[1] Chapter 2 (Sections 2.1 to 2.3).

**Week 2:** Conditional probability, Independent events, Baye's theorem.
[1] Sections 2.4, and 2.5.

**Week 3:** Random Variables, Discrete and continuous random variables, Probability distribution functions discrete random variables, p.m.f, c.d.f, Expectation, Moments, Moment generating functions of discrete random variables.
[1] Chapter 3 (Sections 3.1 to 3.4).

**Week 4:** Probability Distribution functions continuous random variables, p.d.f, c.d.f, Expectation, Moments, Moment generating functions of continuous random variables.
[1] Sections 4.1, and 4.2.

**Week 5:** Discrete distribution**:** Binomial distribution and its m.g.f., Discrete distribution**:** Poisson and its m.g.f.
[1] Chapter 3 (Sections 3.5, and 3.7).

**Week 6:** Geometric distribution, Continuous distribution**:** Normal and its m.g.f.
[1] Chapter 3 (Sections 3.2, and 3.6, excluding negative binomial distribution)
[1] Chapter 4 (Section 6.5)

**Weeks 7 and 8:** Exponential distribution and its "memoryless" property, Bivariate distribution, conditional distribution and marginal distribution, Covariance**,** Correlation and regression.
[1] Chapter 4 (Section 4.3 Pages 193 to 196), and Chapter 5 (Sections 5.1 Exclude more than two variables, 5.2, and 5.3 omit bivariate normal distribution)

**Week 9:** Weak law of large numbers and central limit theorem for independent and identically distributed random variables.
[1] Chapter 6 (Section 6.2).

**Weeks 10 and 11:** Definitions of random sample, Parameter and statistic, Sampling distribution of mean, Standard error of sample mean, Mean, variance of random sample from a normal population, Mean, variance of random sample from a finite population.
[2] Chapter 8 (Sections 8.1 to 8.3).

**Week 12:** Chi-square distribution, t- distribution and F- distribution.
[1] Chapter 6 (Section 6.4).

**Weeks 13 and 14:** Test of hypotheses based on a single sample.
[1] Chapter 9 (Sections 9.1 to 9.4).

**Facilitating the Achievement of Course Learning Outcomes**

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Determine moments and distribution function using moment generating functions. | (i) Each topic to be explained with examples.<br>(ii) Students to be involved in discussions and encouraged to ask questions.<br>(iii) Students to be given | • Student presentations.<br>• Participation in discussions.<br>• Assignments and class tests.<br>• Mid-term |
| 2. | Learn about various discrete and continuous probability distributions.<br>Know about correlation and | | |

| | | | |
|---|---|---|---|
| | regression for two variables, weak law of large numbers and central limit theorem. | homework/assignments.<br>(iv) Students to be encouraged to give short presentations. | examinations.<br>• End-term examinations. |
| 3. | Test validity of hypothesis, using Chi-square, F and t-tests, respectively in sampling distributions. | | |

**Keywords:** Bayes theorem, Binomial, Poisson, Geometric, Normal and exponential distributions, Central limit theorem, Chi-square distribution, F-distribution and t-distribution, Correlation and regression for two variables, Moments and moment generating functions, Weak law of large numbers.

# DSE-1 (ii): Discrete Mathematics

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. The mathematics of modern computer science is built almost entirely on discrete math, in particular Boolean algebra and graph theory. The aim of this course is to make the students aware of the fundamentals of lattices, Boolean algebra and graph theory.

**Course Learning Outcomes:** The course will enable the students to understand:
- i) Learn about partial ordering of sets and various types of lattices.
- ii) Learn about Boolean algebra and switching circuits with Karnaugh maps.
- iii) Know about basics of graph theory and four color map problem.

**Unit 1: Partial Ordering**
Definition, Examples and properties of posets, Maps between posets, Algebraic lattice, Lattice as a poset, Duality principle, Sublattice, Hasse diagrams; Products and homomorphisms of lattices, Distributive lattice, Complemented lattice.

**Unit 2: Boolean Algebra and Switching Circuits**
Boolean algebra, Boolean polynomial, CN form, DN form; Simplification of Boolean polynomials, Karnaugh diagram; Switching circuits and its applications, Finding CN form and DN form.

**Unit 3: Graph Theory**
Graphs, Subgraph, Complete graph, Bipartite graph, Degree sequence, Euler's theorem for sum of degrees of all vertices, Eulerian circuit, Seven bridge problem, Hamiltonian cycle, Adjacency matrix, Dijkstra's shortest path algorithm (improved version), Digraphs; Definitions and examples of tree and spanning tree, Kruskal's algorithm to find the minimum spanning tree; Planar graphs, Coloring of a graph and chromatic number.

**Reference:**
1. Rosen, Kenneth H. (2011). *Discrete Mathematics and its Applications with Combinatorics and Graph Theory* (7th ed.). McGraw-Hill Education Private Limited. Special Indian Edition.

**Additional Readings:**
- i. Goodaire, Edgar G. & Parmenter, Michael M. (2011). *Discrete Mathematics with Graph Theory* (3rd ed.). Pearson Education (Singapore) Pvt. Ltd. Indian Reprint.
- ii. Hunter, David J. (2017). *Essentials of Discrete Mathematics* (3rd ed.). Jones & Bartlett Learning, LLC.
- iii. Lidl, Rudolf & Pilz, Günter (1998). *Applied Abstract Algebra* (2nd ed.). Springer. Indian Reprint 2014.

**Teaching plan (DSE-1 (ii): Discrete Mathematics):**
**Week 1:** Definition, Examples and properties of posets, Maps between posets.
[1] Chapter 7 (Sections 7.5, and 7.6, Pages 493 to 511)
**Weeks 2 and 3:** Algebraic lattice, Lattice as a poset, Duality principle, Sublattice, Hasse diagrams; Products and homomorphisms of lattices, Distributive lattice, Complemented lattice.

[1] Chapter 7 (Section 7.6, Pages 511 to 521)
**Week 4:** Boolean algebra, Boolean polynomial, CN form, DN form.
[1] Chapter 10 (Sections 10.1, and 10.2, Pages 687 to 698)
**Week 5:** Simplification of Boolean polynomials, Karnaugh diagram.
[1] Chapter 10 (Section 10.4, Pages 704 to 718)
**Week 6:** Switching circuits and its applications, Finding CN form and DN form.
[1] Chapter 10 (Section 10.3, Pages 698 to 704)
**Week 7:** Graphs, Subgraph, Complete graph, Bipartite graph,
[1] Chapter 8 (Sections 8.1, and 8.2, Pages 527 to 549)
**Week 8:** Degree sequence, Euler's theorem for sum of degrees of all vertices.
[1] Chapter 8 (Sections 8.3, and 8.4, Pages 549 to 571)
**Week 9:** Eulerian circuit, Seven bridge problem, Hamiltonian cycle.
[1] Chapter 8 (Section 8.5, Pages 571 to 584)
**Week 10:** Adjacency matrix, Dijkstra's shortest path algorithm (improved version), Digraphs.
[1] Chapter 8 (Section 8.6, Pages 585 to 595)
**Week 11 and 12:** Definitions and examples of tree and spanning tree.
[1] Chapter 9 [Sections 9.1 (Pages 623 to 634), 9.3, and 9.4 (Pages 649 to 673)]
**Week 13:** Kruskal's algorithm to find the minimum spanning tree.
[1] Chapter 9 (Section 9.5, Pages 675 to 680)
**Week 14:** Planar graphs, coloring of a graph and chromatic number.
[1] Chapter 8 (Section 8.7, and 8.8, Pages 595 to 613).

**Facilitating the Achievement of Course Learning Outcomes**

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Learn about partial ordering of sets and various types of lattices. | (i) Each topic to be explained with examples.<br>(ii) Students to be involved in discussions and encouraged to ask questions.<br>(iii) Students to be given homework/assignments.<br>(iv) Students to be encouraged to give short presentations. | • Student presentations.<br>• Participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• End-term examinations. |
| 2. | Learn about Boolean algebra and switching circuits with Karnaugh maps. | | |
| 3. | Know about basics of graph theory and four color map problem. | | |

**Keywords:** CN and DN form, Digraphs and planar graphs, Distributive and complemented lattice, Eulerian circuit, Karnaugh diagram, Posets and its lattices, Seven bridge problem, Switching circuits and its applications.

# Semester-VI

# Skill Enhancement Paper

## SEC-4: Statistical Software: R

**Total Marks: 100** (Theory: 38, Internal Assessment: 12, and Practical: 50)
**Workload:** 2 Lectures, 4 Practicals (per week) **Credits:** 4 (2+2)
**Duration:** 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

**Course Objectives:** The purpose of this course is to help you begin using **R**, a powerful free software program for doing statistical computing and graphics. It can be used for exploring and plotting data, as well as performing statistical tests.

**Course Learning Outcomes:** This course will enable the students to:

i) Be familiar with **R** syntax and use **R** as a calculator.
ii) Understand the concepts of objects, vectors and data types.
iii) Know about summary commands and summary table in **R**.
iv) Visualize distribution of data in **R** and learn about normality test.
v) Plot various graphs and charts using **R**.

**Unit 1: Getting Started with R - The Statistical Programming Language**
Introducing **R**, using **R** as a calculator; Explore data and relationships in **R**; Reading and getting data into **R:** combine and scan commands, viewing named objects and removing objects from **R**, Types and structures of data items with their properties, Working with history commands, Saving work in **R**; Manipulating vectors, Data frames, Matrices and lists; Viewing objects within objects, Constructing data objects and their conversions.

**Unit 2: Descriptive Statistics and Tabulation**
Summary commands**:** Summary statistics for vectors, Data frames, Matrices and lists; Summary tables.

**Unit 3: Distribution of Data**
Stem and leaf plot, Histograms, Density function and its plotting, The Shapiro−Wilk test for normality, The Kolmogorov−Smirnov test.

**Unit 4: Graphical Analysis with R**
Plotting in **R:** Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts, Bar charts; Copy and save graphics to other applications.

**References:**

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Gardener, M. (2012). *Beginning* **R:** *The Statistical Programming Language*, Wiley Publications.

**Additional Reading:**

i. Verzani, John (2014). *Using R for Introductory Statistics* (2nd ed.). CRC Press, Taylor & Francis Group.

## Practicals to be done in the Computer Lab using Statistical Software R:

[1] Chapter 14 (Exercises 1 to 3). [2] Relevant exercises of Chapters 2 to 5, and 7.

**Note:** The practical may be done on the database to be downloaded from https://data.gov.in/

### Teaching Plan (Theory of SEC-4: Statistical Software: R):

**Weeks 1 to 3:** Introducing **R**, using **R** as a calculator; Explore data and relationships in **R**, Reading and getting data into **R:** Combine and scan commands, viewing named objects and removing objects from **R**, Types and structures of data items with their properties, Working with history commands, Saving work in **R**.

[1] Chapter 14 (Sections 14.1 to 14.4).
[2] Chapter 2.

**Weeks 4 and 5:** Manipulating vectors, Data frames, Matrices and lists; Viewing objects within objects, Constructing data objects and their conversions.

[2] Chapter 3.

**Weeks 6 to 8:** Summary commands**:** Summary statistics for vectors, Data frames, Matrices and lists; Summary tables.

[2] Chapter 4.

**Weeks 9 to 11:** Stem and leaf plot, Histograms, Density function and its plotting, The Shapiro–Wilk test for normality, The Kolmogorov-Smirnov test.

[2] Chapter 5.

**Weeks 12 to 14:** Plotting in **R:** Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts, Bar charts; Copy and save graphics to other applications.

[1] Chapter 14 (Section 14.7).
[2] Chapter 7.

### Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Be familiar with **R** syntax and use **R** as a calculator. Understand the concepts of objects, vectors and data types. | (i) Topics to be explained with illustrations using **R** software.<br>(ii) Students to be given homework/assignments.<br>(iii) Students to be encouraged to look for new applications. | • Presentations and participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• Practical examinations.<br>• End-term examinations. |
| 2. | Know about summary commands and summary table in **R**. | | |
| 3. | Visualize distribution of data in **R** and learn about normality test. | | |
| 4. | Plot various graphs and charts using **R**. | | |

**Keywords:** Objects, Vectors, Data types, Summary commands, Shapiro–Wilk test, Bar charts.

# Mathematics: DSE–2

## Any *one* of the following:

**DSE-2 (i): Numerical Methods**
**DSE-2 (ii): Differential Equations**

## DSE-2 (i): Numerical Methods

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course Objectives:** The goal of this paper is to acquaint students for the study of certain algorithms that uses numerical approximation for the problems of solving polynomial equations, transcendental equations, linear system of equations, interpolation, and problems of ordinary differential equations.

**Course Learning Outcomes:** After completion of this course, students will be able to:

i) Find the consequences of finite precision and the inherent limits of numerical methods.
ii) Appropriate numerical methods to solve algebraic and transcendental equations.
iii) Solve first order initial value problems of ordinary differential equations numerically using Euler methods.

**Unit 1: Errors and Roots of Transcendental and Polynomial Equations**
Floating point representation and computer arithmetic, Significant digits; Errors**:** Roundoff error, Local truncation error, Global truncation error, Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

**Unit 2: Algebraic Linear Systems and Interpolation**
Gaussian elimination method (with row pivoting), Gauss–Jordan method; Iterative methods**:** Jacobi method, Gauss–Seidel method; Interpolation**:** Lagrange form, Newton form, Finite difference operators, Gregory–Newton forward and backward difference interpolations, Piecewise polynomial interpolation (Linear and quadratic).

**Unit 3: Numerical Differentiation, Integration and ODE**
Numerical differentiation**:** First and second order derivatives, Richardson extrapolation method; numerical integration**:** Trapezoidal rule, Simpson's rule; Ordinary differential equation**:** Euler's method, Modified Euler's methods **(**Heun's and midpoint).

**References:**

1. Chapra, Steven C. (2018). *Applied Numerical Methods with* MATLAB *for Engineers and Scientists* (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). *Applied Numerical Analysis Using* MATLAB. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). *Numerical Methods for Scientific and Engineering Computation* (6th ed.). New Age International Publishers. Delhi.

**Additional Reading:**

i. Bradie, Brian (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

## Teaching Plan (DSE-2(i): Numerical Methods):

**Weeks 1 and 2:** Floating point representation and computer arithmetic, Significant digits; Errors**:** Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.

[2] Chapter 1 (Sections 1.2.3, 1.3.1, and 1.3.2).

[3] Chapter 1 (Sections 1.2, 1.3).

**Week 3 and 4:** Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

[2] Chapter 2 (Sections 2.1 to 2.3).

[3] Chapter 2 (Sections 2.2 and 2.3).

**Week 5:** Gaussian elimination method (with row pivoting), Gauss–Jordan method; Iterative methods**:** Jacobi method, Gauss–Seidel method.

[2] Chapter 3 (Sections 3.1, and 3.2), Chapter 6 (Sections 6.1, and 6.2)

[3] Chapter 3 (Sections 3.2, and 3.4)

**Week 6:** Interpolation**:** Lagrange form, and Newton form.

[2] Chapter 8 (Section 8.1).

[3] Chapter 4 (Section 4.2)

**Weeks 7 and 8:** Finite difference operators, Gregory–Newton forward and backward difference interpolations.

[3] Chapter 4 (Sections 4.3, and 4.4).

**Week 9:** Piecewise polynomial interpolation**:** Linear and quadratic.

[2] Chapter 8 [Section 8.3 (8.3.1, and 8.3.2)].

[1] Chapter 18 (Sections 18.1 to 18.3)

**Weeks 10, 11 and 12:** Numerical differentiation**:** First and second order derivatives, Richardson extrapolation method; Numerical integration**:** Trapezoidal rule, Simpson's rule.

[2] Chapter 11 [Sections 11.1 (11.1.1, 11.1.2 and 11.1.4), and 11.2 (11.2.1, and 11.2.2)]

**Weeks 13 and 14:** Ordinary differential equations**:** Euler's method, Modified Euler's methods **(**Heun's and midpoint).

[1] Chapter 22 (Sections 22.1, 22.2 (up to Page 583) and 22.3).

### Facilitating the Achievement of Course Learning Outcomes

| Unit No. | Course Learning Outcomes | Teaching and Learning Activity | Assessment Tasks |
|---|---|---|---|
| 1. | Find the consequences of finite precision and the inherent limits of numerical methods. | (i) Each topic to be explained with examples.<br>(ii) Students to be involved in discussions and encouraged to ask questions.<br>(iii) Students to be given homework/assignments.<br>(iv) Students to be encouraged to give short presentations. | • Presentations and participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• End-term examinations. |
| 2. | Appropriate numerical methods to solve algebraic and transcendental equations. | | |
| 3. | Solve first order initial value problems of ordinary differential equations numerically using Euler methods. | | |

**Keywords:** Bisection method, Euler's method, Gauss–Jordan method, Gauss–Seidel method, Jacobi method, Newton–Raphson method, Regula–Falsi method, Richardson extrapolation method, Secant method and Simpson's rule.

# DSE-2 (ii): Differential Equations

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)
**Workload:** 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hours.

**Course objectives:** The course aims at introducing ordinary and partial differential equations to the students and finding their solutions using various techniques with the tools needed to model complex real-world situations.

**Course learning outcomes:** The course will enable the students to:

i) Solve ODE's and know about Wronskian and its properties.
ii) Method of variation of parameters and total differential equations.
iii) Solve linear PDE's of first order.
iv) Understand Lagrange's and Charpit's methods for solving nonlinear PDE's of first order.

**Unit1: Ordinary Differential Equations**
First order exact differential equations including rules for finding integrating factors, First order higher degree equations solvable for $x$, $y$, $p$ and Clairut's equations; Wronskian and its properties, Linear homogeneous equations with constant coefficients; The method of variation of parameters; Euler's equations**;** Simultaneous differential equations; Total differential equations.

**Unit 2: Linear Partial Differential Equations**
Order and degree of partial differential equations, Concept of linear partial differential equations, Formation of first order partial differential equations, Linear partial differential equations of first order and their solutions.

**Unit 3: Non-linear Partial Differential Equations**
Concept of non-linear partial differential equations, Lagrange's method, Charpit's method, classification of second order partial differential equations into elliptic, parabolic and hyperbolic through illustrations only.

**References:**

1. Ross, Shepley L. (1984). *Differential Equations* (3rd ed.). John Wiley & Sons, Inc.
2. Sneddon, I. N. (2006). *Elements of Partial Differential Equations*, Dover Publications. Indian Reprint.

**Additional Readings:**

i. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pvt. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
ii. Brannan, James R., Boyce, William E., & McKibben, Mark A. (2015). *Differential Equations*: *An Introduction to Modern Methods and Applications* (3rd ed.). John Wiley & Sons, Inc.

**Teaching Plan (DSE-2 (ii): Differential Equations):**

**Weeks 1 and 2:** First order exact differential equations including rules for finding integrating factors.
[1] Chapter 2 (Section 2.1).
**Weeks 3 and 4:** First order higher degree equations solvable for $x$, $y$, $p$ and Clairut's equations.
[1] Chapter 2 (Sections 2.2, and 2.3).
**Weeks 5 and 6:** Wronskian and its properties, Linear homogeneous equations with constant coefficients.
[1] Chapter 4 (Sections 4.1, and 4.2).
**Week 7:** The method of variation of parameters, Euler's equations.

[1] Sections 4.3, and 4.4.

**Week 8:** Simultaneous differential equations, Total differential equations.

[2] Chapter 1 (Sections 2, 3, 5, and 6)

**Week 9:** Order and degree of partial differential equations, Concept of linear partial differential equations, Formation of first order partial differential equations.

[2] Chapter 2 (Section 1.2).

**Weeks 10 and 11:** Statement of Theorem 2 with applications, Linear partial differential equations of first order and their solutions.

[2] Chapter 2 (Sections 3, 4, 5, and 6).

**Week 12:** Statements of Theorems 4, 5, and 6 with applications, Concept of non-linear partial differential equations, Lagrange’s method.

[2] Chapter 2 (Sections 7, 8, and 9).

**Weeks 13 and 14:** Charpit’s method, Classification of second order partial differential equations into elliptic, Parabolic and hyperbolic through illustrations only.

[2] Chapter 2 (Section 10), and Chapter 3 (Sections 1, and 5).

## Facilitating the Achievement of Course Learning Outcomes

<table>
<tr><th>Unit No.</th><th>Course Learning Outcomes</th><th>Teaching and Learning Activity</th><th>Assessment Tasks</th></tr>
<tr><td>1.</td><td>Solve ODE’s and know about Wronskian and its properties. Method of variation of parameters and total differential equations.</td><td rowspan="3">(i) Each topic to be explained with examples.<br>(ii) Students to be involved in discussions and encouraged to ask questions.<br>(iii) Students to be given homework/assignments.<br>(iv) Students to be encouraged to give short presentations.</td><td rowspan="3">• Presentations and participation in discussions.<br>• Assignments and class tests.<br>• Mid-term examinations.<br>• End-term examinations.</td></tr>
<tr><td>2.</td><td>Solve linear PDE’s of first order.</td></tr>
<tr><td>3.</td><td>Understand Lagrange’s and Charpit’s methods for solving nonlinear PDE’s of first order.</td></tr>
</table>

**Keywords:** Charpit's method, Clairut’s equations, Euler’s equations, Lagrange’s method, Wronskian and its properties.

## Acknowledgments

The following members were actively involved in drafting the LOCF syllabus of B.A. (Programme), University of Delhi.

**Head**

- C.S. Lalitha, Department of Mathematics

**Coordinator**

- Hemant Kumar Singh, Department of Mathematics

**Committee Members**

- Sarla Bhardwaj (Dr. Bhim Rao Ambedkar College)
- Anuradha Gupta (Delhi College of Arts and Commerce)
- A.R. Prasannan (Maharaja Agrasen College)